Nonlinear three-dimensional magnetoconvection around magnetic flux tubes

G. J. J. Botha

Centre for Fusion, Space and Astrophysics, Physics Department, University of Warwick, Coventry CV4 7AL, UK

G.J.J.Botha@warwick.ac.uk

A. M. Rucklidge

Department of Applied Mathematics, University of Leeds, Leeds LS2 9JT, UK

A.M.Rucklidge@leeds.ac.uk

N. E. Hurlburt

Lockheed Martin Solar and Astrophysics Laboratory, Organization ADBS Building 252, Palo Alto, CA 94304, USA

hurlburt@lmsal.com

ABSTRACT

Magnetic flux in the solar photosphere forms concentrations from small scales, such as flux elements, to large scales, such as sunspots. This paper presents a study of the decay process of large magnetic flux tubes, such as sunspots, on a supergranular scale. 3D nonlinear resistive magnetohydrodynamic numerical simulations are performed in a cylindrical domain, initialised with axisymmetric solutions that consist of a well-defined central flux tube and an annular convection cell surrounding it. As the nonlinear convection evolves, the annular cell breaks up into many cells in the azimuthal direction, allowing magnetic flux to slip between cells away from the central flux tube (turbulent erosion). This lowers magnetic pressure in the central tube and convection grows inside the tube, possibly becoming strong enough to push the tube apart. A remnant of the central flux tube persists with nonsymmetric perturbations caused by the convection surrounding it. Secondary flux concentrations form between convection cells away from the central tube. Tube decay is dependent on the convection around the tube. Convection cells forming inside the tube as time-dependent outflows will remove magnetic flux. (This is most pronounced for small tubes). Flux is added to the tube when flux caught in the surrounding convection is pushed toward it. The tube persists when convection inside the tube is sufficiently suppressed by the remaining magnetic field. All examples of persistent tubes have the same effective magnetic field strength, consistent with the observation that pores and sunspot umbrae all have roughly the same magnetic field strength.

Subject headings: convection — magnetohydrodynamics (MHD) — Sun: interior — Sun: surface magnetism — sunspots

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1. Introduction

The appearance of magnetic field on the visible surface of the Sun ranges through many length scales. Magnetic elements surface in the photosphere and are convected to the edges of the convection cells or granules. These elements have a diameter of up to approximately 100 km, a field strength of 1.5 kG and a lifetime of a few minutes (Bello González et al. 2009; Zhang et al. 1998; de Wijn et al. 2009). Where flux elements congregate, as at the boundaries of granules and supergranules, pores form that have diameters between 2 and 4 Mm and field strengths of approximately 2 kG (Bray & Loughhead 1964; Zwaan 1992). These pores have lifetimes of typically less than a day (Keppens 2001). As magnetic flux accumulates in a pore, it can grow large enough to form a sunspot with a penumbra. Sunspots can be up to 50 Mm in diameter and have life times of a few hours to several weeks. When a sunspot disintegrates, its flux remnants are convected to the polar regions where they are observed as polar elements with diameters of approximately 300 km, field strengths of above 1 kG and lifetimes from several hours to days (Tsuneta et al. 2008; de Wijn et al. 2009). Excellent reviews on all aspects of sunspots are by Solanki (2003) and Thomas & Weiss (2004).

Sunspot decay is often expressed in terms of the temporal evolution of sunspot area. A linear decay rate indicates diffusive processes, while a quadratic decay rate points to erosion of the sunspot by the surrounding convection. Hathaway & Choudhary (2008) looked at the decay rate of sunspot groups and from this calculated the decay rate per individual sunspot. They found that, on average, each sunspot decayed at a rate independent of the area of the spot, i.e. a diffusive process. In contrast, Petrovay & Van Driel-Gesztelyi (1997) looked at individual sunspots and found a parabolic decay rate suggesting erosion by the turbulent convection surrounding the spot. Observations of the outer penumbral boundary shows fluctuations around an average position, indicating an interplay between convective motion and the sunspot's magnetic field (Kubo et al. 2008).

A reasonable approximation for the structure of a pore is a cylindrical magnetic flux tube with inflowing convection surrounding it. Observations of the Sun show patches of strong downflows around the flux concentrations (Hirzberger 2003; Rimmele 2004; Stangl & Hirzberger 2005). Around the edge of pores hair-like striations have been observed with an azimuthal wavelength smaller than the surrounding granular convection (Scharmer et al. 2002; Berger et al. 2004). These striations are believed to be magnetoconvective downflow lanes. Needle-like structures have been observed surrounding pores with an internal flow towards the pore and a downflow at the end near the flux concentration (Sankarasubramanian & Rimmele 2003). A pore growing in size due to accumulated flux may evolve a rudimentary penumbral structure. This proto-penumbra is transitory in nature and may oscillate between penumbral-like filaments and elongated granules (Dorotovič et al. 2002), decay (Sobotka et al. 1999) or evolve into a fully developed sunspot penumbra (Keppens & Martínez Pillet 1996). The formation of a fully formed penumbra around sunspots is usually abrupt, with a sudden change of the magnetic field direction from vertical to inclined (Rucklidge et al. 1995; Yang et al. 2003).

Unlike the flow surrounding pores, a sunspot is surrounded by a moat cell that consists of a surface flow that are flowing predominantly away from the sunspot. The moat flow sometimes exhibits azimuthal structure, with spoke-like lanes of converging flow which have a higher average concentration of outwardly moving magnetic features (Shine & Title 2001; Hagenaar & Shine 2005). Hurlburt & Rucklidge (2000) found in a numerical study of idealised axisymmetric flux tubes in cylinders that a steady collar flow with converging flow at the top of the convection cell is always established around the flux tube. Following Parker's hypothesis (Parker 1979) that a sunspot is a cluster of flux tubes held together by a collar flow, they speculated that sunspots must also have a collar flow. The well-known annular ring of inward moving penumbral grains surrounding a sunspot umbra (Thomas & Weiss 2004) may be explained by three different mechanisms: the first is the emergence of flux tubes through the photosphere (Schlichenmaier et al. 1998); the second by moving patterns caused by granular magnetoconvection in an oblique magnetic field (Hurlburt et al. 1996); and thirdly the presence of a collar flow around the umbra. Observations that the inward radial movement survives the breakup of the penumbra in a decaying sunspot while the umbra stays intact (Deng et al. 2007) support the presence of a collar flow. Helioseismic measurements also support the concept of a collar flow that ensures the integrity of the umbral flux tube (Gizon & Birch 2005; Tong 2005; Zhao et al. 2010). Measurements of p-modes show that underneath the Evershed flow in the penumbra, there exist a converging flow as well as a downflow up to a depth of approximately 3 Mm. Below these flows there is an outflow that extends to more than 30 Mm from the sunspot axis. However, this result is ambiguous because the flow does not appear in f-mode measurements, which give only an outflow to a depth of at least 10 Mm that corresponds to the moat flow on the surface (Gizon & Birch 2005). In addition to this, Moradi et al. (2010) have shown that different methods of helioseismology give different flow patterns in the subsurface of a sunspot. As a result, the interaction of solar acoustic waves with strong magnetic flux concentrations is actively studied through observations (Braun & Birch 2008), numerical simulations (Shelyag et al. 2009; Parchevsky & Kosovichev 2009) and analytical investigations (Gordovskyy et al. 2009; Jain et al. 2009).

Numerical simulations of magnetoconvection in the upper layer of the solar convection zone follow two complementary strategies. The first is to include as many physical processes as are practical given the numerical constraints. Ionisation, radiative energy transfer and a numerical domain that typically stretches from the temperature minimum in the chromosphere down to a few Mm beneath the visible surface of the Sun have been included in models of three-dimensional (3D) granulation in the quiet Sun (Stein & Nordlund 2006), two-dimensional (2D) flux sheets in an initially unmagnetised, stratified, convecting atmosphere heated from below (Leka & Steiner 2001), 3D pores surrounded by granulation (Cameron et al. 2007), flux emergence of a semi-toroidal loop introduced into a purely hydrodynamic background in statistical equilibrium (Cheung et al. 2010), as well as 3D sunspot umbrae with their penumbrae embedded in the surrounding granulation (Rempel et al. 2009a,b). The second strategic approach to numerical simulations, which is the one we follow in this paper, is to simplify the physics and explore magnetoconvection in this parameter space. As such, our upper domain boundary is 0.5 Mm below the visible surface of the Sun. This allows us to consider the interplay between magnetic field and convection without the complications of sharp gradients due to the stratification. It also means that we cannot say anything about penumbral formation.

The results presented in this paper is a generalisation to three dimensions of 2D (axisymmetric) nonlinear magnetoconvection (Hurlburt & Rucklidge 2000). The axisymmetric results are characterised by a magnetic flux tube at the central axis, with uniform temperature and magnetic field strength inside the tube. Around the flux tube convection cells form concentric rings with the inner ring converging onto the flux tube at the top of the domain, forming a collar flow around the flux tube. The magnetic flux bundle and the convection rings around it are essentially time independent. The larger the radii of the cylindrical numerical domain, the more counter-rotating convection rings form around the central flux bundle. By decreasing the aspect ratio of the cylindrical numerical domain, time dependence was introduced (Botha et al. 2006). For intermediate radii, a small convection ring forming the collar flow establishes itself around the top of the flux bundle, with a dominating counterflow at its outside border. The collar flow is periodically destroyed by the counterflow and reforms. During this process the magnetic field expands radially in the absence of the collar flow and is pushed back into a tight flux bundle on the central axis upon the reformation of the collar flow. Botha et al. (2008) showed that when the numerical domain is rotated, a Rankine vortex forms: the magnetic flux tube rotates as a rigid body while sheared azimuthal flow (i.e. a free vortex) forms in the surrounding convection cells.

In this paper the robustness of a magnetic flux tube surrounded by 3D nonsymmetric convection is presented. It is shown that a remnant of the original flux tube persists in spite of these nonsymmetric perturbations, with secondary flux concentrations forming around the remnant. The secondary concentrations form due to magnetic flux escaping from the central flux bundle, and their strength depends on an interplay between the strength of the magnetic field in the simulation and the vigour of the convection. In the simulations we have used magnetic field strengths of $Q = 32$, 100 and 250, where Q is the Chandrasekhar number. The numerical domain is a cylindrical wedge with an aspect ratio of 3.

This study of 3D nonlinear convection was preceded by initialising the numerical simulations with a time independent (nonlinear) 2D solution and perturbing the plasma in the azimuthal direction (Botha et al. 2007). The linear evolution of these perturbations showed that steady and oscillating instabilities with a preferred azimuthal number formed in the convecting flow close to the outer edge of the flux tube. A concise summary and expansion of these results are given in Section 4 before the discussion moves on to the nonlinear magnetoconvection results that form the bulk of this paper.

The paper first discusses the model in the next section, and then describes the design of the numerical experiments and their initialisation (Section 3). The evolution of the linear azimuthal modes is summarised in Section 4, before the nonlinear results are presented in Section 5. In the discussion we consider the influence of an increasing Chandrasekhar number (Q) on the results, as well as the effect of the azimuthal width of the numerical domain (Section 5.1). The paper concludes with a discussion and short summary of the results (Sections 6 and 7).

2. Model

The initial temperature and density profiles in the vertical (z) direction are given by the polytrope

$$T = T_0(1+\theta z), \tag{1}$$
$$\rho = \rho_0(1+\theta z)^m, \tag{2}$$

with the 0 subscript defining the quantity at the top of the box ($z=0$), θ is the initial temperature gradient, and m is the polytropic index. The equations for fully compressible, nonlinear three-dimensional (3D) magnetoconvection are

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\mathbf{v}\rho) \tag{3}$$

$$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v}\cdot\nabla\mathbf{v} + \theta(m+1)\hat{\mathbf{z}} + \frac{\sigma\zeta_0 K^2 Q}{\rho}\mathbf{j}\times\mathbf{B} - \frac{1}{\rho}\nabla P + \frac{\sigma K}{\rho}\left(\nabla^2\mathbf{v} + \frac{1}{3}\nabla\nabla\cdot\mathbf{v}\right) \tag{4}$$

$$\frac{\partial T}{\partial t} = -\mathbf{v}\cdot\nabla T - (\gamma-1)T\nabla\cdot\mathbf{v} + \frac{\gamma K}{\rho}\nabla^2 T + \frac{\zeta_0}{\rho}\left|\mathbf{j}\right|^2 \tag{5}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \zeta_0 K\nabla^2\mathbf{B} - \nabla\psi \tag{6}$$

with the auxiliary equations

$$P = \rho T, \qquad \mathbf{j} = \nabla\times\mathbf{B}. \tag{7}$$

The variable ψ is introduced to enforce the condition $\nabla\cdot\mathbf{B}=0$ (Dedner et al. 2002), and it evolves through

$$\frac{\partial\psi}{\partial t} = -c_h^2\nabla\cdot\mathbf{B} - \frac{c_h^2}{c_p^2}\psi, \tag{8}$$

where c_h and c_p are constants. We use the following notation: γ the ratio of specific heats; σ the Prandtl number; ζ_0 the magnetic diffusivity ratio at $z=0$; and Q is the Chandrasekhar number defined as

$$Q = \frac{(Bd)^2}{\mu\rho\eta\nu}, \tag{9}$$

where B is the magnetic field, d is the depth of the domain, μ the magnetic permeability, η the magnetic diffusivity and ν the kinematic viscosity. The dimensionless thermal conductivity K is related to the Rayleigh number R in the following way:

$$R = \theta^2(m+1)\left[1 - \frac{(m+1)(\gamma-1)}{\gamma}\right]\frac{(1+\theta/2)^{2m-1}}{\sigma K^2} \tag{10}$$

R is a measure of the importance of buoyancy forces compared to viscous forces in the middle of the layer. All the other symbols have their usual meaning. The physical quantities are dimensionless, with the length scaled proportional to the depth of the numerical domain, velocities scaled proportional to the sound speed at the top of the domain and temperature, magnetic field, density, and pressure all scaled proportional to their initial values at the top of the numerical domain, so $T_0 = 1$ and $\rho_0 = 1$.

The numerical implementation of the model was developed specifically for these types of calculations (Hurlburt & Rucklidge 2000). The cylindrical grid is in the shape of a wedge with dimensions

$$0 \le r \le \Gamma, \quad 0 \le \phi \le 2\pi/M_\phi, \quad 0 \le z \le 1, \tag{11}$$

with $z=0$ at the top of the domain. Sixth-order compact finite differencing is used in the vertical (r,z) plane and the domain has a spectral azimuthal (ϕ) direction. The level of dealiasing is increased toward the central axis to maintain grid uniformity. For time evolution we use a fourth-order modified (explicit) Bulirsch-Stoer integration technique. The number of grid points in the radial direction is typically $64 \times \Gamma$, in the vertical direction 64, and in the azimuthal direction between 64 and 512.

The boundary condition in the azimuthal direction is periodic. At the central axis we impose regularity conditions and the domain has a slippery, perfectly conducting, thermally insulating outer wall. The way that we impose on-axis regularity conditions constrains the value of M_ϕ to be at least 4. Both the top and bottom boundaries are impenetrable, with a vertical magnetic field and with their respective temperatures fixed. The boundaries are described in detail in Botha et al. (2006).

3. Initialisation and design of numerical experiments

All the numerical results in the paper were obtained with the following parameter values: $R = 10^5$, $\sigma = 1$, $\zeta_0 = 0.2$, $\theta = 10$, $\gamma = 5/3$ and $m = 1.495$. The values of the Chandrasekhar number Q and the azimuthal width M_ϕ were varied and will be reported throughout the paper. These physical parameter values were chosen to describe the solar convection zone from a depth of approximately 0.5 Mm below the visible surface of the Sun to a depth of approximately 6 Mm (Botha et al. 2006). The numerical domain is a cylindrical wedge of one unit deep and a radius of $\Gamma = 3$, so that the simulations are on a supergranular scale. It is worth pointing out that the value of the Prandtl number used here is much larger than the value of σ in the upper layer of the solar convection zone. In axisymmetric simulations we have used $\sigma = 1$ and $\sigma = 0.3$ (Botha et al. 2008). The result was the same qualitatively, displaying more vigorous convection in the case of lower σ. The same was found for the 3D numerical simulations presented in this paper. The stronger convection forced smaller time steps and denser numerical grids, so that we used $\sigma = 1$ for expediency in all the results presented here.

The three-dimensional simulations are initialised with an axisymmetric, or two-dimensional, solution as shown in Figure 1. It consists of a well defined flux tube at the central axis with a convection cell around it, forming an annular collar flow that contains the magnetic flux at the central axis. This solution was obtained by starting 2D simulations with a uniform field and a velocity perturbation. Galloway, Proctor & Weiss (1978) showed that for an incompressible Boussinesq fluid with vertical top and bottom magnetic boundaries, the magnetic flux is concentrated almost entirely at the central axis. Previous numerical results show that this is also true for compressible fluids, with the width of the numerical box determining the number of convection cells forming around the flux bundle (Hurlburt & Rucklidge 2000). We have taken care to allow only one convection cell to form, by adapting the value of Q to the size of the domain. The stronger magnetic field in the case of higher Q values causes a wider flux bundle to form at the centre, in this way allowing us to regulate the remaining space in which convection forms.

The linear phase of the 3D simulations were started by perturbing the vertical component of the velocity in the azimuthal direction. The perturbation took the form of a cosine wave with wavelength equal to the azimuthal width of the numerical domain. By changing the width of the wedge, we were able to measure the growth rates of different wavelengths (Botha et al. 2007). The solution with the largest linear growth-rate was then chosen to continue into the nonlinear regime. In order to test the robustness of the nonlinear results obtained in this manner, we also initialised 3D simulations with a uniform vertical magnetic field. Flux separation occurred and eventually these results were indistinguishable from the nonlinear results obtained as described above.

4. Linear azimuthal evolution

In the linear phase the solution forms a static axisymmetric standing wave inside the magnetic flux tube, and one annular convection cell around the tube, as shown in Figure 2(a). The density profile of the solution is hardly perturbed from the polytrope, because the atmosphere is close to adiabatic, i.e. m is close to $1/(\gamma-1)$. In order to obtain the location of the linear eigenmodes, we take the azimuthal perturbation of the physical quantities, as shown in Figure 2(b). This shows that the linear modes are situated at the edge of the magnetic flux tube. The azimuthal size of the linear modes is related to the size of the magnetic flux tube. Figure 3 shows that for small tube radii the linear azimuthal lengths are short. As the tube radius increases, its influence on the azimuthal length scale decreases. The result is a well-defined length scale in the limit of large radii. This is a geometric effect, with the azimuthal length of the local linear mode dependent on the curvature of the tube radius that it samples.

Botha et al. (2007) determined that the eigenfunctions of the linear modes forming around the magnetic flux tubes are convective in nature. As in that paper, we found linear modes with steady growth and modes with oscillating growth, with the steadily growing modes occurring at low values of M_ϕ.

5. Nonlinear azimuthal evolution

As the solution evolves from the linear into the nonlinear regime, the annular convection cell around the magnetic flux tube breaks up into many cells. The absence of a uniform collar flow allows the magnetic field to expand into the areas between the convection cells, which in turn lowers the magnetic field strength near the central axis ($r = 0$). Lower magnetic strength inside the flux tube means that low levels of convection are allowed to form, the maximum amplitude of which depends on the magnetic field strength in the numerical domain.

For $Q = 32$ the axisymmetric initial condition is a narrow vertical flux tube at the central axis. In this case a slight increase in tube radius allows for a relatively large decrease in magnetic field strength. It follows that after a small amount of magnetic field has moved between the convection cells surrounding the central flux tube, convection forms inside the tube that grows and becomes strong enough to break it up. The end result is magnetic flux between the convection cells with only a flux tube remnant at the central axis. This final state is shown in Figure 4 at a time when the nonlinear magnetoconvection is well established.

In the case of $Q = 100$, the axisymmetric initial condition consists of a central magnetic flux tube that is wider than the case for $Q = 32$. As soon as the annular convection cell around the flux tube starts to break into many cells, magnetic flux moves between the cells radially away from the central axis. The relative change in the magnetic field at the central axis is less than for $Q = 32$ and weaker convection forms inside the flux tube. As a consequence, it takes longer for the convection to erode the magnetic field away from the central axis. Eventually a steady nonlinear state forms with magnetic flux between the convection cells and with a reduced central magnetic flux tube. Figure 5 shows the result for $Q = 100$. As the solution evolves through time, the magnetic flux is pushed around by the convection. In particular, the flux forming the central tube is buffeted by strong irregular convection. This leads to the central flux tube changing shape, with radial tendrils forming temporarily between the convection cells surrounding the flux tube. Figure 6 shows radial tendrils forming between strong convection cells that have their genesis just inside the edge of the flux tube. At time 673.59 the convection cells have grown strong enough to separate flux from the central flux tube and push flux into radial tendrils between them.

Figure 7 shows the result for $Q = 250$ when the nonlinear magnetoconvection is well established. Compared to the axisymmetric initial condition, the radius of the magnetic flux tube is only slightly larger when nonlinear magnetoconvection is established. This means the weak convection forming inside the flux tube is not strong enough to push magnetic flux away from the central axis. The edge of the magnetic flux tube is buffeted by the convection around it, as in the case for $Q = 100$, but fewer radial tendrils form between the convection cells and those that do form are weaker than in the case when $Q = 100$. In order to see the same breakup of the magnetic tube as for $Q = 32$, one has to enlarge the radius of the numerical domain (Γ) so that the magnetic flux has more room to disperse between the convection cells and in the process lowers the magnetic field strength at the central axis.

A quantitative measure of the breakup of the central magnetic flux tube is provided by defining the radial profile of the magnetic field as

$$|\bar{B}(r)| = \frac{1}{nz.n\phi} \sum^{nz} \sum^{n\phi} \sqrt{B_r^2 + B_\phi^2 + B_z^2}\ , \quad (12)$$

where nz and $n\phi$ are the number of data points in the vertical and azimuthal directions respectively.

This gives an indication of the size of the magnetic field in the radial direction, i.e. the magnetisation of the plasma as a function of the radius. The magnetic field $|\bar{B}| = |\bar{B}(r)|$ is then multiplied by a factor of $\sqrt{Q}$ in order to facilitate comparison of the magnetic field strength between the different sets of parameter values. Figures 4(d), 5(d) and 7(d) give the radial profiles of $\sqrt{Q}|\bar{B}|$ for $Q = 32$, $Q = 100$ and $Q = 250$ respectively at different times during the numerical simulations. All of them show a well-defined magnetic flux tube at the central axis at the axisymmetric initial condition. The annular ring around the central flux tube breaks up into many cells in the azimuthal direction for all values of Q. In the case of $Q = 32$ and $Q = 100$, two concentric circles of cells form around the magnetic flux tube. These cells have an upflow next to the inner flux tube as well as next to the outer boundary (Figures 4 and 5). This convection pattern allows magnetic flux to move between the cells in the radial direction and to be captured between the cells at a radial position of approximately $r = 1.7$. The value of the magnetic field at these positions are similar for $Q = 32$ and $Q = 100$, as shown in Figures 4(d) and 5(d). These figures also show the relatively large decrease of magnetic field at the central axis for $Q = 32$ and the lesser decrease for $Q = 100$. In the case of $Q = 250$ only one ring of convection cells forms around the magnetic flux tube, flowing toward the magnetic flux bundle at the top of the domain (Figure 7). Figure 7(d) shows that the magnetic flux tube radius increases slightly, as only a small amount of flux moves between the convection cells. The magnetic field inside the flux tube decreases less than in the case for $Q = 100$, so that weaker convection forms inside the flux tube and the tube stays intact throughout the simulation run.

Where strong flows carve into the central magnetic tube, the magnetic field is pushed together and the field strength experiences a local peak value. This can be seen in Figure 7(a) for $Q = 250$. At the edge of the flux tube, indentations into the tube is caused by strong convection pushing against the side of the tube. The magnetic field is also stronger at these indentations than at the protrusions between them, where the magnetic field pushes into weaker convection flows.

The interplay between the central magnetic flux tube and the convection surrounding it can be seen in Figure 8, where the time evolution of $|\bar{B}| = |\bar{B}(r)|$ is shown for the different values of Q. Figure 8(a) shows that for $Q = 32$ at time 600, the convection inside the magnetic flux tube becomes strong enough to push a significant part of the magnetic flux away from the central axis. In contrast, $Q = 250$ in Figure 8(c) has a magnetic field that is strong enough to keep the magnetic flux tube intact, in spite of weak convection that forms inside the flux tube. In the case of $Q = 100$, Figure 8(b) shows that weak convection starts at time 550. Eventually the convection grows strong enough to push magnetic flux away from the central flux tube, so that at time 1000 the nonlinear magnetoconvection with $Q = 32$ and with $Q = 100$ look similar.

The magnetic flux that escapes from the central flux tube tends to congregate at strong downflows between convection cells where the convection is converging. In the region where the convection dominates, this is between the convection cells where strong downflows occur. Figure 9 shows the region of convection in green, and the locations of strong magnetic field in blue. One clearly observes the magnetic flux remnant at the central axis through the absence of convection as well as the magnetic field concentrated there. The downward plumes in the convection can be seen with their associated strong magnetic field concentrations. It is noticeable that the magnetic field strength in the plumes is maximum in the top half of the numerical domain, while the downward velocities reaches their maximum in the bottom half of the domain. The chaotic convection near the bottom boundary in Figure 9 is a consequence of our impenetrable boundary condition and is not relevant to the current discussion. Where the magnetic field peaks, either at the central axis or in the downward plumes, an azimuthal current forms a ring around the magnetic field concentration. This is shown in the vertical planes of Figures 4, 5 and 7 for all values of Q.

Figure 10 shows the close relation between the convection and the temperature in the solution. Everywhere an upflow occurs, the plasma is hot relative to the surrounding temperature. As a result, the temperature perturbation on every plane shows clearly the forms of the convection cells, as can be seen in the top part of the numerical domain. In the case of $Q = 100$, the upflow created by the two concentric rings of convection cells is observable in Figure 10 next to the magnetic flux tube as well as next to the outer boundary. At the plumes between convection cells where strong downflow occurs, the plasma is cool relative to the surrounding temperature. Figure 10 shows the downflows together with their associated lower temperature near the bottom of the domain.

In previous axisymmetric simulations of a central flux tube surrounded by convection, an annular cell flowing towards the flux tube at the top of the numerical domain was always present to keep the flux tube intact (Hurlburt & Rucklidge 2000). In the absence of this collar flow the magnetic flux tube spread out radially until the collar flow was restored (Botha et al. 2006). We observe the same phenomenon in three dimensions. Figure 7 shows one inward flowing collar cell that keeps the magnetic flux tube confined to the central axis, with very little magnetic field present in the convection. In contrast, the outward flowing cells of $Q = 32$ and $Q = 100$ in Figures 4 and 5 allow magnetic flux to escape from the flux tube. It is interesting that the flux tube is not completely destroyed in these cases. A remnant of the tube is left at the central axis, which becomes more prominent as the value of Q increases.

The Alfvén speed in the model is defined as $c_A^2 = \sigma\zeta_0 K^2 Q B^2/\rho$. Figure 9 shows that the highest values of the magnetic field occurs at the top of the downflows. This is true for both $Q = 32$ (Figure 4) and $Q = 100$ (Figures 5 and 9). Consequently these are also the locations for the highest values of c_A in the numerical domain, since the density increases as one moves downwards.

5.1. Effect of wedge width

The effect of the wedge width on the nonlinear results is investigated by doubling the azimuthal angle of the numerical domain from $M_\phi = 8$ to $M_\phi = 4$ and then continuing the simulation run. Figure 11 shows the result for $Q = 32$. In this case the convection cells merge from two concentric circles, each with eight cells in the azimuthal direction, to one concentric circle consisting of four cells in the azimuthal direction. Figure 12 shows how initially part of the magnetic field is trapped between the convection cells, which then moves towards the central axis as the convection pattern changes from two concentric circles to one. Three instances from Figure 12 are plotted in Figure 11(d), with the final state showing that most of the magnetic field are being pushed towards the central axis.

The initial state of this simulation is similar to that depicted in Figure 4. Here the convection forms two concentric circles of eight cells each in the azimuthal direction, with orientation such that the inner radial cells flow radially outward and the outer cells radially inward at the top of the domain, as can be seen in Figures 4(b) and (c). During the simulation the inner cells become weaker while the outer cells grow stronger (Figure 12). The final state in the wider wedge is shown in Figure 11. Figure 11(c) shows the large convection cells that formed, with strong flow moving inward at the top of the domain. Figure 11(b) shows a region on the boundary between the large convection cells. Here one finds weaker convection and a magnetic field that is pushed between the two cells away from the central axis, so that the width of the central flux tube is larger here than at positions where the strong convection forces the magnetic flux against the central axis. This is clearly visible in Figures 11(a), (b) and (c).

For $Q = 100$ and $Q = 250$ the number of cells in the radial and azimuthal directions stays the same when the numerical domain is widened. Figure 13 shows the simulation result when $Q = 250$. This behaviour can be explained by considering the geometry of the solution. A convection cell naturally wants to maintain a shape where its radial size is approximately the same as its azimuthal size. For low values of Q, the magnetic flux tube has a small radius and the numerical domain containing the convection a large one. In a narrower wedge, as in Figure 4 for $Q = 32$ and $M_\phi = 8$, the size of convection cells is limited by constraints in the azimuthal direction. This means that two cells form in the radial direction, each of which has approximately the same radial diameter as its azimuthal diameter. By doubling the wedge width, as in Figure 11, this constraint is lifted so that the azimuthal width becomes approximately the same as the radial width of the convective area in the numerical domain. Consequently one cell forms that fit into the radial as well as the azimuthal directions. For $Q = 250$ the magnetic flux tube fills half the radius of the domain, while the other half contains the convection (Figure 13). In this case the radial width of one convection cell fits eight times into one full azimuthal rotation. As a consequence, doubling the width of the numerical domain has no effect on the number of cells forming around the magnetic flux bundle. We conclude that as long as the radial width of the convection area (r_c) is smaller or of equal size to the azimuthal width of the numerical domain, i.e.

$$r_c \leq \frac{2\pi r_p}{M_\phi}, \tag{13}$$

where r_p is the radius of the magnetic flux tube, then the width of the domain will not restrict the formation of cells in the convection around the flux tube.

6. Discussion

This paper can be thought of as an exploration of two physical processes: flux separation and turbulent erosion. Flux separation occurs when turbulent eddies expel magnetic flux to their borders where the flux concentration builds up (Weiss 1966), while turbulent erosion is the process when convection cells push against magnetic flux concentrations and allow the flux to escape between them away from the area of high concentration (Simon & Leighton 1964).

The 2D axisymmetric solutions used as initial conditions for the 3D numerical simulations presented here, have a well-defined magnetic flux tube at the central axis with an annular convection cell around it (Figure 1). Irrespective of the radius of the domain, i.e. the number of counter-rotating convection cells that fit into the domain, almost all the magnetic flux in the solution gathers at the central flux bundle (Hurlburt & Rucklidge 2000). This final state is maintained indefinitely.

The initial magnetic flux bundles have radii that increase with increasing values of Q. The dimensional values for these radii are 4.75 Mm for $Q = 32$, 6.22 Mm for $Q = 100$ and 7.86 Mm for $Q = 250$. The dimensional magnetic field strength can in principle be calculated from the Chandrasekhar number as defined in (9), but the measurement of μ, η and ν are uncertain in the convection zone. Alternatively, one can compare the effective magnetic field strength as given by $\sqrt{Q}|\bar{B}|$, with $|\bar{B}|$ defined in (12). Figures 4(d), 5(d) and 7(d) show that the value of the effective magnetic field strength, once the flux tube has settled down, are similar (about 40), irrespective of the Q value. Furthermore, the value of $\sqrt{Q}|\bar{B}|$ stays reasonably constant for the width of each magnetic flux bundle, as was shown by Hurlburt & Rucklidge (2000). This relatively homogeneous umbral magnetic field is a typical feature of sunspots (Solanki 2003).

As soon as a third dimension is introduced to the axisymmetric solution, the annular convection cell breaks up in the azimuthal direction into convection cells that have approximately equal radial and azimuthal diameters (Figures 4, 5 and 7). The dimensions of the convection cells were discussed in Section 5.1. The convection cells that push against the central magnetic flux tube push some of the magnetic flux between them, which reduces the magnetic pressure inside the flux tube. This allows weak magnetoconvection cells to form inside the tube, which can grow strong enough to break the flux tube up through flux separation (Figure 8). A remnant of the original flux tube remains at the central axis, while the rest of the magnetic flux is captured between the convection cells in the numerical domain.

Most of the magnetic flux between cells gather at the locations of the strongest downflows (Figure 9). These downflowing plumes are well known from Cartesian simulations of magnetoconvection (Cattaneo et al. 2003). They exhibit strong magnetic flux concentrations in the top layers of the numerical domain. The velocity increases as the flow moves downward along the plume. The magnetic flux gathering at these locations are not strong enough to influence the convection pattern around them. As a result, the flux concentrations are easily manipulated and destroyed by the convection surrounding them, similar to the final destruction of solar pores as observed by Sobotka et al. (1999).

The central magnetic flux bundle is weakened by flux slipping away from it between the convection cells that exist around the flux bundle. This process is highly dynamic, with temporary radial tendrils forming between the cells, to be destroyed again when the magnetoconvection pushes them back to the central axis. Petrovay & Moreno-Insertis (1997) solved the 2D axisymmetric diffusion equation with a diffusivity dependent on the magnetic field, to show that a magnetic flux tube decays from its outer edge to its centre, in the process forming a sharp gradient at its edge. They also found that a fraction of the magnetic flux tube always remains. This idealised result will be true when small convection cells surround a large magnetic flux bundle. In the results we presented in this paper, eight convection cells (Figure 4) as well as four convection cells (Figure 11) leave a flux bundle remnant at the central axis, provided that the radius of the numerical domain is small in order to limit the amount of convection cells forming.

Unlike the idealised diffusion result of Petrovay & Moreno-Insertis (1997), the decay of the central flux bundle is not a steady process, but highly dependent on the nonlinear magnetoconvection surrounding the flux bundle. As convection cells form inside the magnetic flux bundle, they push the flux away from the central axis (Figure 8). The opposite process occurs when convection cells are destroyed next to the central flux bundle. In this case the magnetic flux that was captured between the cells is pushed towards the central axis, thus increasing the radius of the magnetic flux bundle (Figure 12). Thus the decay of the central magnetic flux tube is dominated by magnetoconvection (Figure 8) and we cannot fit a linear or parabolic decay rate to these numerical results. Another possible influence of the convection on the magnetic structure occurs when weak convection forms inside the magnetic flux bundle. If the convection is too weak to destroy the integrity of the flux bundle, it only increases the diameter of the flux bundle slightly, as shown in Figure 8(c).

All the simulations were done in a numerical domain in the shape of a cylindrical wedge. The axis of the domain fixed the centre so that translation of the flux tube as a whole is impossible, and the axis is a natural location for magnetic field to concentrate. These are the main differences from performing the calculations in a Cartesian domain. Different wedge widths, $M_\phi = 8$ in Figures 4, 5 and 7 and $M_\phi = 4$ in Figures 11 and 13, show that the sizes of the convection cells around the magnetic flux tube are determined by the distance between the edge of the flux tube and the outer boundary. Provided that the azimuthal width is large enough to accommodate a convection cell that fits into this radial length, the cylindrical wedge gives similar answers to a Cartesian domain.

7. Conclusion

This paper presents numerical simulations of magnetic flux tubes in a 3D cylindrical domain, solving the nonlinear resistive magnetohydrodynamic equations. The simulations were initialised with an axisymmetric (2D) solution consisting of a central magnetic flux tube and one annular convection cell around it. The solution was then perturbed with an azimuthal velocity perturbation.

Linear modes form in the azimuthal direction, situated on the border between the central flux tube and the convection cell. During the linear stage of the developing azimuthal perturbation, the wavelength of the fastest growing mode depends on the radius of the central magnetic flux tube. For small tube radii the wavelength is small, while it increases as the tube radius increases. However, the dependence of the mode length on the tube radius decreases as the tube radius increases, so that there exist a well-defined length scale in the limit of large radii.

When the nonlinear three-dimensional convection develops, the annular cell breaks up into many cells. Magnetic flux slips between the cells away from the central flux tube. This process is known as turbulent erosion. The magnetic pressure in the central tube becomes less and convection grows inside the flux tube that can become strong enough to push the tube apart. A remnant of the central flux tube persists, undergoing nonsymmetric perturbations caused by the convection surrounding it. The size of the central tube remnant and its perturbations both depend on the interplay between the magnetic field and the convection. For $Q = 32$, strong convection forms at the edge of the flux tube that pushes flux away from the central tube. For $Q = 100$ weaker convection forms inside the tube that takes longer to reached the same final state as when $Q = 32$. For $Q = 250$, the convection that forms inside the tube is too weak to break the integrity of the tube. In all cases, the effective magnetic field strength $\sqrt{Q}|\bar{B}|$ in the remaining magnetic flux tube is approximately the same, regardless of the flux tube radius. This is consistent with the axisymmetric results of Hurlburt & Rucklidge (2000) and the observation that pores and sunspot umbrae of all sizes all have roughly the same magnetic field strength (Zwaan & Brants 1981; Solanki 2002).

The decay of the tube is dependent on the convection around it. The convection can remove flux from the tube when new convection cells form inside the flux tube, or add flux to the tube when convection cells next to the flux tube are destroyed and flux caught between the convection cells is pushed back to the central tube. The convection can also change the shape of the central flux tube by pushing against the magnetic flux so that some of the flux is pressed in between convection cells. In this way during the simulations, radial tendrils develop between the convection cells that surround the central flux tube. These are time dependent, forming and disappearing as the convection pattern around the central flux tube changes.

Secondary flux concentrations form between the convection cells away from the central tube. This occurs at temporary downflowing plumes that form between convection cells where the convection converges.

The conclusion from the work presented in this paper is that the decay of a central flux tube is dictated by the nonlinear magnetoconvection surrounding it. The formation and destruction of convection cells around the flux tube can add or subtract magnetic flux from the tube. As such, the flux tube's decay rate does not fit a simple law. This is true when all the different parameters are considered together as well as for each individual numerical run. A remnant of the central flux tube always survives, with secondary flux concentrations between the convection cells. The more convection cells form, the more flux is captured between the cells and the central tube becomes less defined.

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REFERENCES

Bello González, N., Yelles Chaouche, L., Okunev, O., & Kneer, F 2009, A&A, 494, 1091

Berger, T. E., Rouppe van der Voort, L. H. M., Löfdahl, M. G., Carlsson, M., Fossum, A., Hansteen, V. H., Marthinussen, E., Title, A., & Scharmer, G. 2004, A&A, 428, 613

Botha, G. J. J., Rucklidge, A. M., & Hurlburt, N. E. 2006, MNRAS, 369, 1611

Botha, G. J. J., Rucklidge, A. M., & Hurlburt, N. E. 2007, ApJ, 662, L27

Botha, G. J. J., Busse, F. H., Rucklidge, A. M., & Hurlburt, N. E. 2008, MNRAS, 387, 1445

Braun, D. C., & Birch, A. C. 2008, Sol. Phys., 251, 267

Bray, J. R., & Loughhead, R. E. 1964, Sunspots, Chapman and Hall, London

Cameron, R., Schüssler, M., Vögler, A., & Zakharov, V. 2007, A&A, 474, 261

Cattaneo, F., Emonet, T., & Weiss, N. O. 2003, ApJ, 588, 1183

Cheung, M. C. M., Rempel, M., Title, A. M., & Schüssler, M 2010, ApJ, 720, 233

de Wijn, A. G., Stenflo, J. O., Solanki, S. K., & Tsuneta, S. 2009 Space Sci. Rev., 144, 275

Dedner, A., Kemm, F., Kröner, D., Munz, C.-D., Schnitzer, T., & Wesenberg, M. 2002, Journal of Computational Physics, 175, 645

Deng, N., Choudhary, D. P., Tritschler, A., Denker, C., Liu, C., & Wang, H. 2007, ApJ, 671, 1013

Dorotovič, I., Sobotka, M., Brandt, P. N., & Simon, G. W., 2002, A&A, 387, 665

Galloway, D. J., Proctor, M. R. E., & Weiss, N. O. 1978, Journal of Fluid Mechanics, 87, 243

Gizon L., Birch A. C., 2005, Living Rev. Solar Phys., 2, 6. Online Article: cited 9/01/2006, http://www.livingreviews.org/lrsp-2005-6

Gordovskyy M., Jain, R., & Hindman, B. W. 2009, ApJ, 694, 1602

Hagenaar, H. J., & Shine, R. A. 2005, ApJ, 635, 659

Hathaway, D. H., & Choudhary, D. P. 2008, Sol. Phys., 250, 269

Hirzberger, J. 2003, A&A, 405, 331

Hurlburt, N. E., Alexander, D. 2002, in COSPAR Colloq. 14, Solar-Terrestrial Magnetic Activity and Space Environment, ed. Wang H. & Xu R. (Oxford: Pergamon), 19

Hurlburt, N. E., Matthews P. C., & Proctor, M. R. E. 1996, ApJ, 457, 933

Hurlburt, N. E., & Rucklidge, A. M. 2000, MNRAS, 314, 793

Jain, R., Hindman, B. W., Braun, D. C., & Birch, A. C. 2009, ApJ, 695, 325

Keppens, R. 2001, in Encyclopedia of Astronomy and Astrophysics, ed. P. Murdin, (Bristol, IOP), 2043, DOI:10.1888/0333750888/2043

Keppens, R., & Martínez Pillet, V. 1996, A&A, 316, 229

Kubo, M., Lites, B. W., Ichimoto, K., Shimizu, T., Suematsu, Y., Katsukawa, Y., Tarbell, T. D., Shine, R. A., Title, A. M., Nagata, S., & Tsuneta, S. 2008, ApJ, 681, 1677

Leka, K. D., & Steiner, O. 2001, ApJ, 552, 354

Moradi, H., Baldner, C., Birch, A. C., Braun, D., Cameron, R., Duvall Jr., T. L., Gizon, L., Haber, D., Hanasoge, S., Jackiewicz, J., Khomenko, E., Komm, R., Rajaguru, P., Rempel, M., Roth, M., Schlichenmaier, R., Schunker, H., Spruit, H., Strassmeier, K., Thompson, M. J., & Zharkov S. 2010, arXiv:0912.4982v1 [astro-ph.SR]

Parchevsky, K. V., & Kosovichev, A. G. 2009, ApJ, 694, 573

Parker, E. N. 1979, ApJ, 230, 905

Petrovay, K, & Moreno-insertis, F. 1997, ApJ, 485, 398

Petrovay, K, & Van Driel-Gesztelyi, L. 1997, Sol. Phys., 176, 249

Rempel, M., Schüssler, M., Cameron, R. H., & Knölker, M. 2009a, Science, 325, 171

Rempel, M., Schüssler, M., & Knölker, M. 2009a, ApJ, 691, 640

Rimmele, T. R. 2004, ApJ, 604, 906

Rucklidge, A. M., Schmidt, H. U., & Weiss, N. O. 1995, MNRAS, 273, 491

Sankarasubramanian, K., & Rimmele, T. 2003, ApJ, 598, 689

Scharmer, G. B., Gudiksen, B. V., Kiselman, D., Löfdahl, M. G., & Rouppe van der Voort, L. H. M. 2002, Nature, 420, 151

Schlichenmaier, R., Jahn, K., & Schmidt, H. U. 1998, A&A, 337, 897

Shelyag, S., Zharkov, S., Fedun, V., Erdélyi, R., & Thompson, M. J. 2009, A&A, 501, 735

Shine, R. A., & Title, A. M. 2001, in Encyclopedia of Astronomy and Astrophysics, ed. Murdin, P., (Bristol: IOP Publishing), vol. 4, p. 3209, doi: 10.1888/0333750888/2038

Simon, G. W., & Leighton, R. B. 1964, ApJ, 140, 1120

Sobotka, M., Vázquez, M., Bonet, J. A., Hanslmeier, A., & Hirzberger, J. 1999, ApJ, 511, 436

Solanki, S. K. 2002, Astron. Nachr., 323, 165

Solanki, S. K. 2003, Astron. Astrophys. Rev., 11, 153

Stangl, S., & Hirzberger, J. 2005, A&A, 432, 319

Stein, R. F., & Nordlund, Å. 2006, ApJ, 642, 1246

Thomas, J. H., & Weiss, N. O. 2004, Annu. Rev. Astron. Astrophys., 42, 517

Tong C. H., 2005, Phil. Trans. R. Soc. A, 363, 2761

Tsuneta, S., Ichimoto, K., Katsukawa, Y., Lites, B. W., Matsuzaki, K., Nagata, S., Orozco Suárez, D., Shimizu, T., Shimojo, M., Shine, R. A., Suematsu, Y, Suzuki, T. K., Tarbell, T. D., & Title, A. M. 2008 ApJ, 688, 1374

Weiss, N. O. 1966, Proc. R. Soc. London A, 293, 310

Yang, G., Xu, Y., Wang, H., & Denker, C. 2003, ApJ, 597, 1190

Zhang, J., Lin, G., Wang, J., Wang, H., & Zirin, H. 1998, Sol. Phys., 178, 245

Zhao, J., Kosovichev, A. G., & Sekii, T. 2010, ApJ, 708, 304

Zwaan, C. 1992 in Sunspots: Theory and Observations, ed. Thomas, J. H., & Weiss, N. O., (Cambridge, UK: Cambridge University Press), 75

Zwaan, C., & Brants, J. J. 1981, in The Physics of Sunspots, ed. Cram, L. E., & Thomas, J. H., (Sunspot, NM: Sacramento Peak Observatory), 210

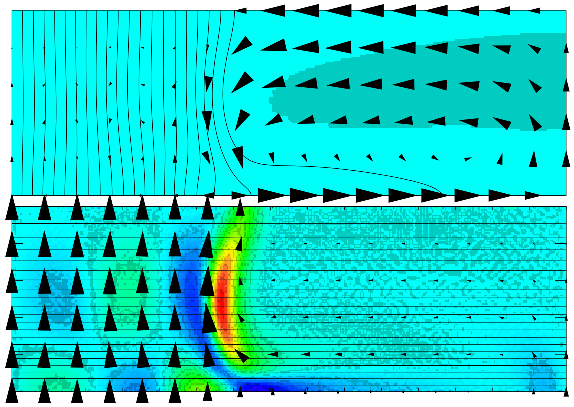
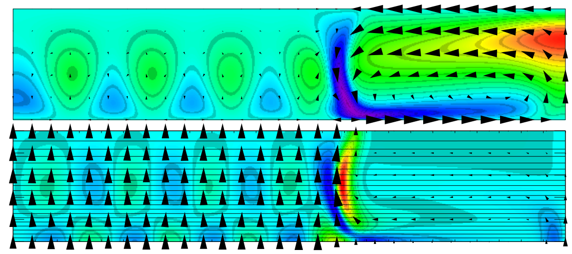


Fig. 1.— Initial axisymmetric state obtained with $Q = 100$ and $\Gamma = 3$. In the top panel the colour (grey in the printed version) represents the temperature fluctuation relative to the unperturbed state, the lines magnetic field and the arrows the velocity field. In the bottom panel the colour (grey in the printed version) represents azimuthal current density j_ϕ in the (r, z) plane, the arrows magnetic field and contours the mass density ρ. The axis is on the left hand edge of each panel.

(a)



(b)

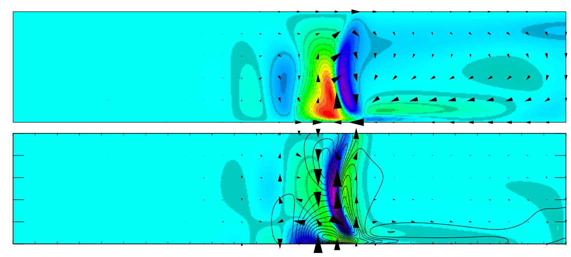


Fig. 2.— 3D state obtained with $Q = 500$ and $\Gamma = 5$. The solution is sampled at the vertical (r, z) plane taken at mid-sector with $M_\phi = 10$. (a) is the full solution and (b) the azimuthal perturbations of the physical quantities. In both (a) and (b) the top panel represents the temperature fluctuation relative to the unperturbed state in colour and the velocity field as arrows. In the bottom panel colour represents azimuthal current density j_ϕ, arrows the magnetic field and contours the mass density ρ. All colour scales are replaced by grey scales in the printed version. The solution forms an axisymmetric standing wave inside the magnetic flux bundle, and one annular convection cell round the flux bundle, as seen in (a). The linear modes are located at the boundary between the magnetic flux bundle and the convection cell, shown in (b). The amplitudes of the linear modes oscillate and this data set was sampled at maximum amplitude. The axis is on the left hand edge of each panel.

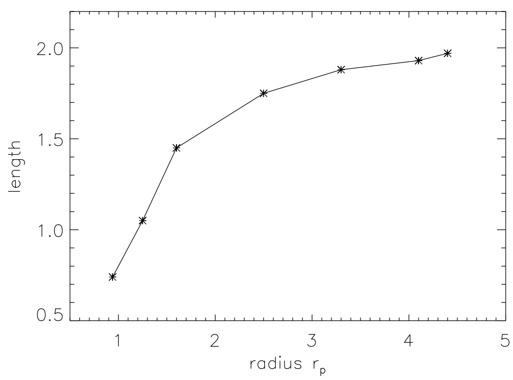


Fig. 3.— Length of linear modes in the azimuthal direction, for different magnetic flux tube radii (r_p). The length is calculated as $2\pi r_p/M_\phi$. The stars are the values measured from our simulations.

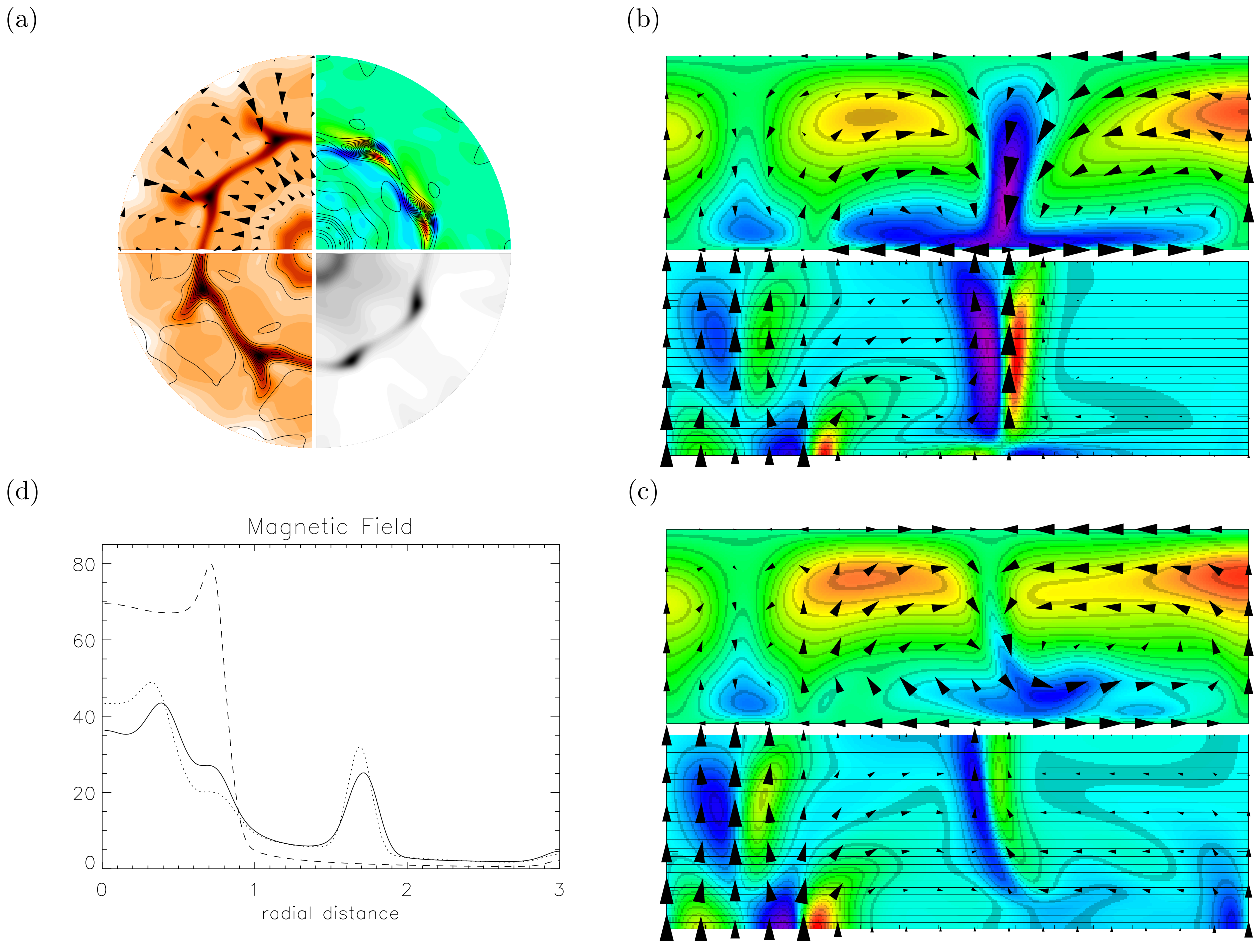


Fig. 4.— Nonlinear solution with $Q = 32$ and $M_\phi = 8$ at time 1006.81. (a) shows the horizontal midplane, and the two vertical planes are sampled at (b) $\phi = 18°$ with max $|\mathbf{B}| = 16.1$ and (c) $\phi = 45°$ with max $|\mathbf{B}| = 12.5$. (d) represents the radial profile of the magnetic field strength $\sqrt{Q}|\bar{B}|$ with $|\bar{B}|$ defined in (12), at three different times. The solid line is at time 1006.8, the dotted line at time 835.7 and the dashed line is the profile of the axisymmetric initial condition at time 328.0. The diagnostics for the horizontal plane (a) are as follows: in the first quadrant the colour represents j_z and the contours B_z; in the second quadrant the colour is the temperature fluctuation relative to the unperturbed state (dark is cold) and arrows are the velocity field; in the third quadrant the colour is the temperature fluctuation relative to the unperturbed state and contours are the mass density; in the fourth quadrant grey is magnetic field strength, with max $|\mathbf{B}| = 15.4$ black and zero white. In the printed version the colour scales in quadrants one, two and three are printed as grey scales. The diagnostics for the two vertical planes are the same as in Figure 2(a). The physics in the two vertical planes are as follows: (b) has $\min(\beta) = 17.2$, $\max(\beta) = 1.6 \times 10^{11}$, $\min(j_\phi) = -118.8$, $\max(j_\phi) = 125.5$, $\min(\tilde{T}) = 0.98$, $\max(\tilde{T}) = 1.01$; (c) has $\min(\beta) = 29.6$, $\max(\beta) = 5.9 \times 10^7$, $\min(j_\phi) = -62.0$, $\max(j_\phi) = 80.3$, $\min(\tilde{T}) = 0.98$, $\max(\tilde{T}) = 1.01$. Here β is the ratio of gas pressure over magnetic pressure. In this solution max(Mach number) = 0.1.

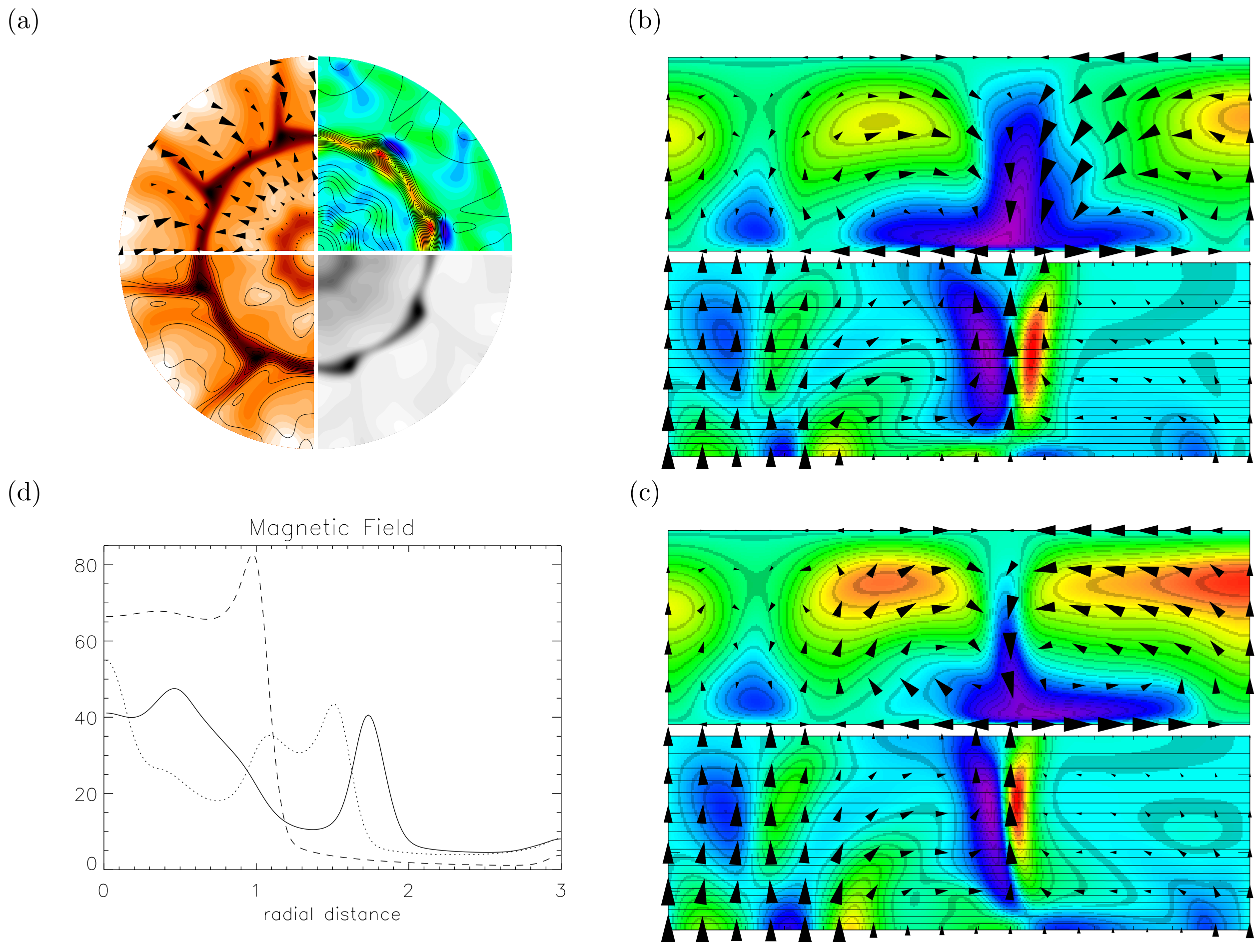


Fig. 5.— Nonlinear solution with $Q = 100$ and $M_\phi = 8$ at time 1043.13. (a) shows the horizontal midplane with $\max|\mathbf{B}| = 9.4$, and the two vertical planes are sampled at (b) $\phi = 15°$ with $\max|\mathbf{B}| = 9.4$ and (c) $\phi = 45°$ with $\max|\mathbf{B}| = 7.9$. (d) represents the radial profile of the magnetic field strength $\sqrt{Q}|\bar{B}|$, with $|\bar{B}|$ defined in (12), at three different times. The solid line is at time 1043.13, the dotted line at time 803.5 and the dashed line is the profile of the axisymmetric initial condition at time 357.1. The diagnostics for the horizontal plane (a) are the same as in Figure 4 and for the two vertical planes as in Figure 2(a). The physics in the two vertical planes are as follows: (b) has $\min(\beta) = 15.8$, $\max(\beta) = 7.4 \times 10^{10}$, $\min(j_\phi) = -54.8$, $\max(j_\phi) = 62.7$, $\min(\tilde{T}) = 0.99$, $\max(\tilde{T}) = 1.01$; (c) has $\min(\beta) = 28.0$, $\max(\beta) = 1.2 \times 10^7$, $\min(j_\phi) = -46.2$, $\max(j_\phi) = 51.7$, $\min(\tilde{T}) = 0.99$, $\max(\tilde{T}) = 1.01$. In this solution max(Mach number) = 0.1.

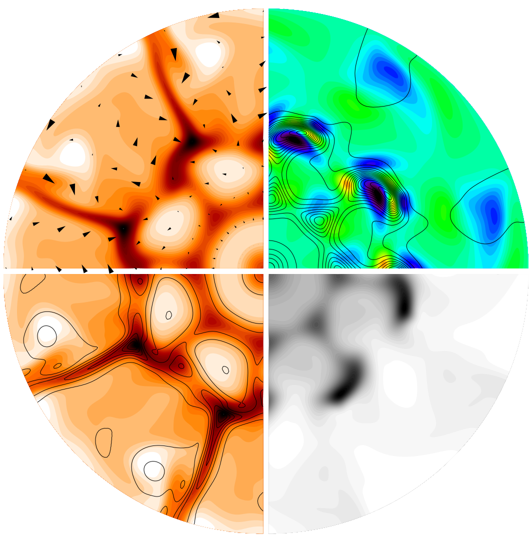


Fig. 6.— Nonlinear solution with $Q = 100$ and $M_\phi = 8$ at time 673.59. This dataset is taken from the same numerical run as Figure 5. The horizontal midplane is shown with $\max|\mathbf{B}| = 8.62$. Flux forming radial tendrils between strong convection cells surrounding the magnetic flux tube is shown. The diagnostics for the horizontal plane are the same as in Figure 4.

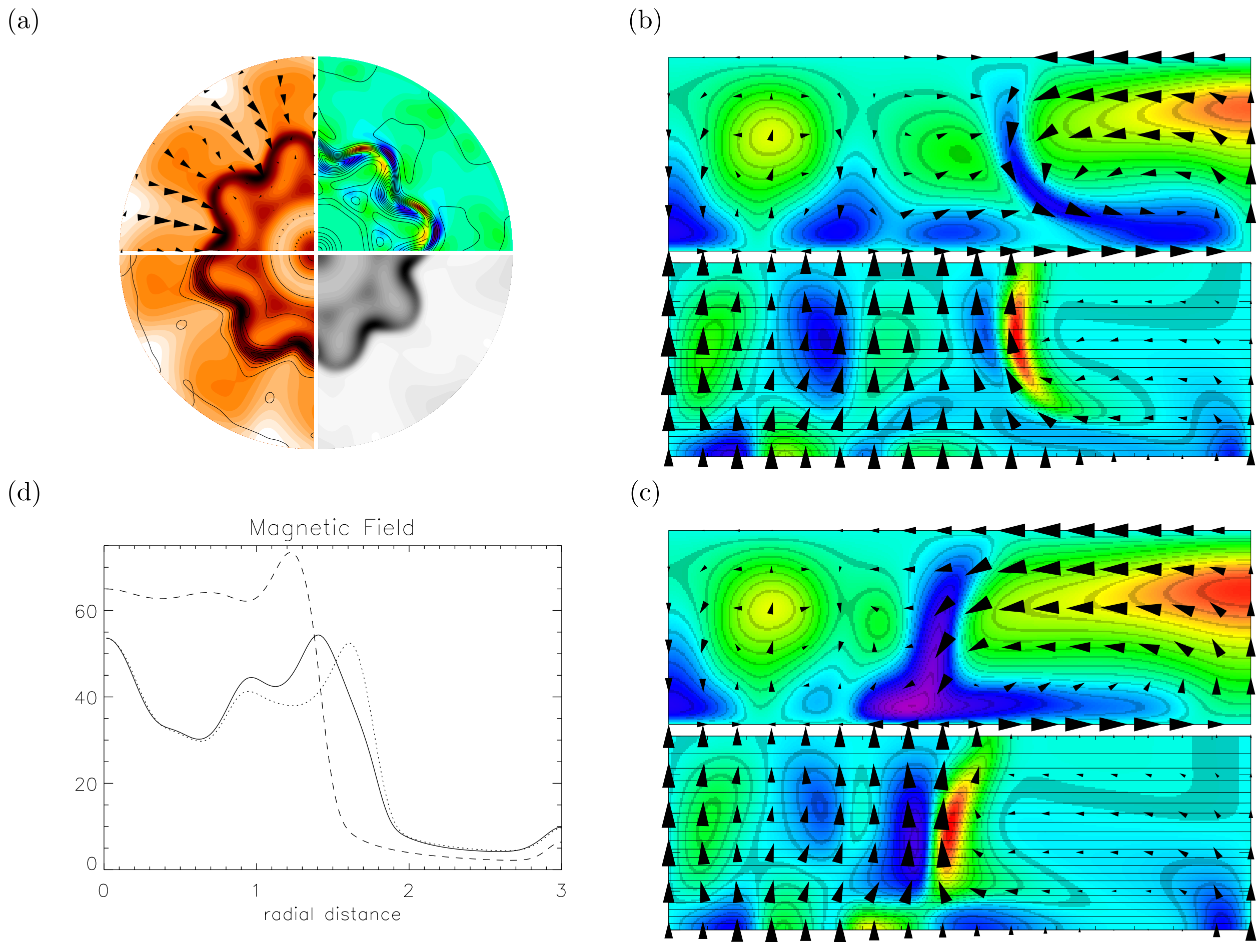


Fig. 7.— Nonlinear solution with $Q = 250$ and $M_\phi = 8$ at time 1034.95. (a) shows the horizontal midplane with $\max|\mathbf{B}| = 6.4$, and the two vertical planes are sampled at (b) $\phi = 10°$ with $\max|\mathbf{B}| = 4.4$ and (c) $\phi = 35°$ with $\max|\mathbf{B}| = 6.4$. (d) represents the radial profile of the magnetic field strength $\sqrt{Q}|\bar{B}|$ with $|\bar{B}|$ defined in (12), at three different times. The solid line is at time 1034.95, the dotted line at time 794.98 and the dashed line is the profile of the axisymmetric initial condition at time 471.34. The diagnostics for the horizontal plane (a) are the same as in Figure 4 and for the two vertical planes as in Figure 2(a). The physics in the two vertical planes are as follows: (b) has $\min(\beta) = 24.2$, $\max(\beta) = 1.2 \times 10^7$, $\min(j_\phi) = -16.3$, $\max(j_\phi) = 32.6$, $\min(\tilde{T}) = 0.99$, $\max(\tilde{T}) = 1.01$; (c) has $\min(\beta) = 13.8$, $\max(\beta) = 7.7 \times 10^{12}$, $\min(j_\phi) = -25.7$, $\max(j_\phi) = 45.5$, $\min(\tilde{T}) = 0.99$, $\max(\tilde{T}) = 1.01$. In this solution max(Mach number) = 0.1.

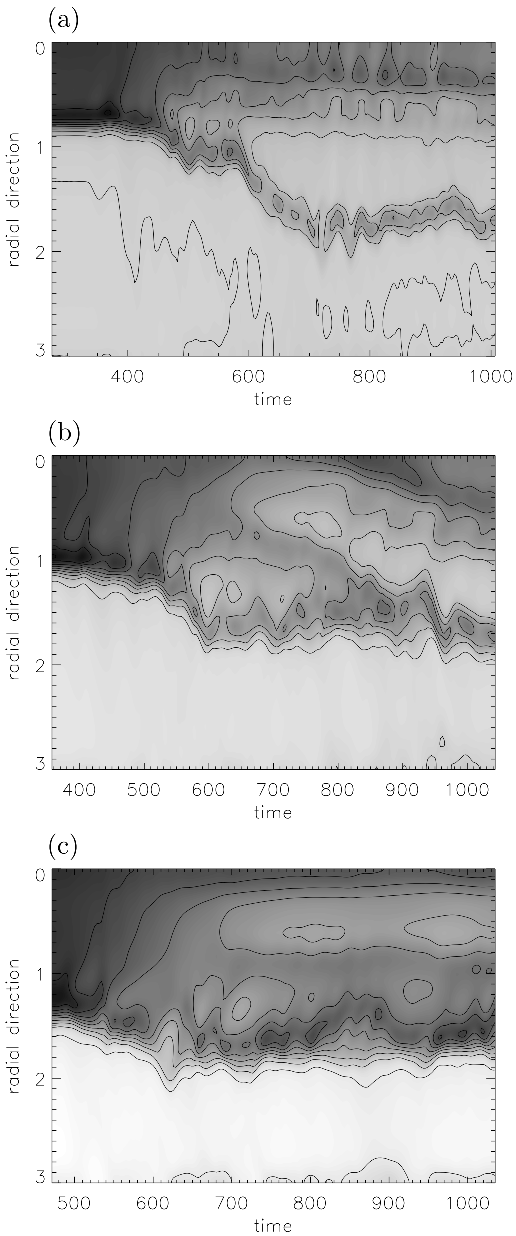


Fig. 8.— The time evolution of the effective magnetic field $\sqrt{Q}|\bar{B}|$ with $|\bar{B}|$ defined in (12). Black represents $\max(\sqrt{Q}|\bar{B}|)$ and white zero. The spatial dimensions are $\Gamma = 3$ and $M_\phi = 8$. The central axis is along the top edge of each panel, and the time begins when the nonaxisymmetric perturbations are introduced. (a) presents $Q = 32$ with $\max(\sqrt{Q}|\bar{B}|) = 86.1$, of which three instances are plotted in Figure 4(d). (b) presents $Q = 100$ with $\max(\sqrt{Q}|\bar{B}|) = 82.7$, of which three instances are plotted in Figure 5(d). (c) presents $Q = 250$ with $\max(\sqrt{Q}|\bar{B}|) = 73.4$, of which three instances are plotted in Figure 7(d). The convection tears magnetic flux away from the the central flux bundle in the cases of $Q = 32$ and $Q = 100$, while the flux bundle stays intact for $Q = 250$.

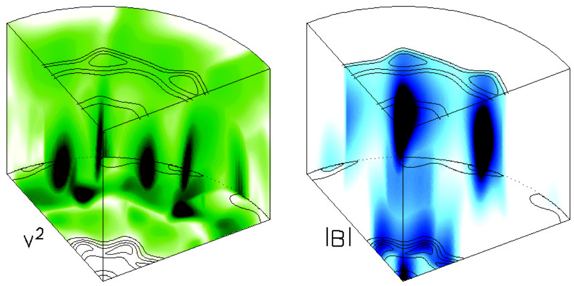


Fig. 9.— A 3D rendering of the kinetic and magnetic energies in the system, with $Q = 100$ and $M_\phi = 8$. The physical parameters are given in Figure 5. On the left hand side is the velocity field (v^2) in green and on the right hand side the magnetic field $|\mathbf{B}|$ in blue. A grey scale is used in the printed version. The contours at the top and bottom panels of the wedge are the vertical magnetic field passing through these planes.

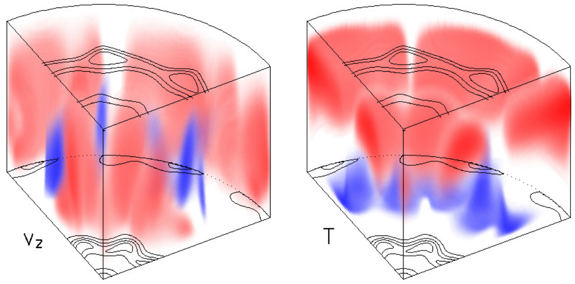


Fig. 10.— The vertical velocity perturbation on each plane ($\tilde{v}_z$) on the left hand side and the temperature perturbation on each plane ($\tilde{T}$) on the right hand side. The velocity is colour coded so that upward flow is red and downward flow blue. For the temperature red shows where the plasma is hot and blue where it is cool relative to the surrounding temperature. In the printed version only the upflow and hot plasma are shown in a grey scale. The physical parameters are as in Figure 5, with $Q = 100$ and $M_\phi = 8$. The contours at the top and bottom panels of the wedge are the vertical magnetic field passing through these planes.

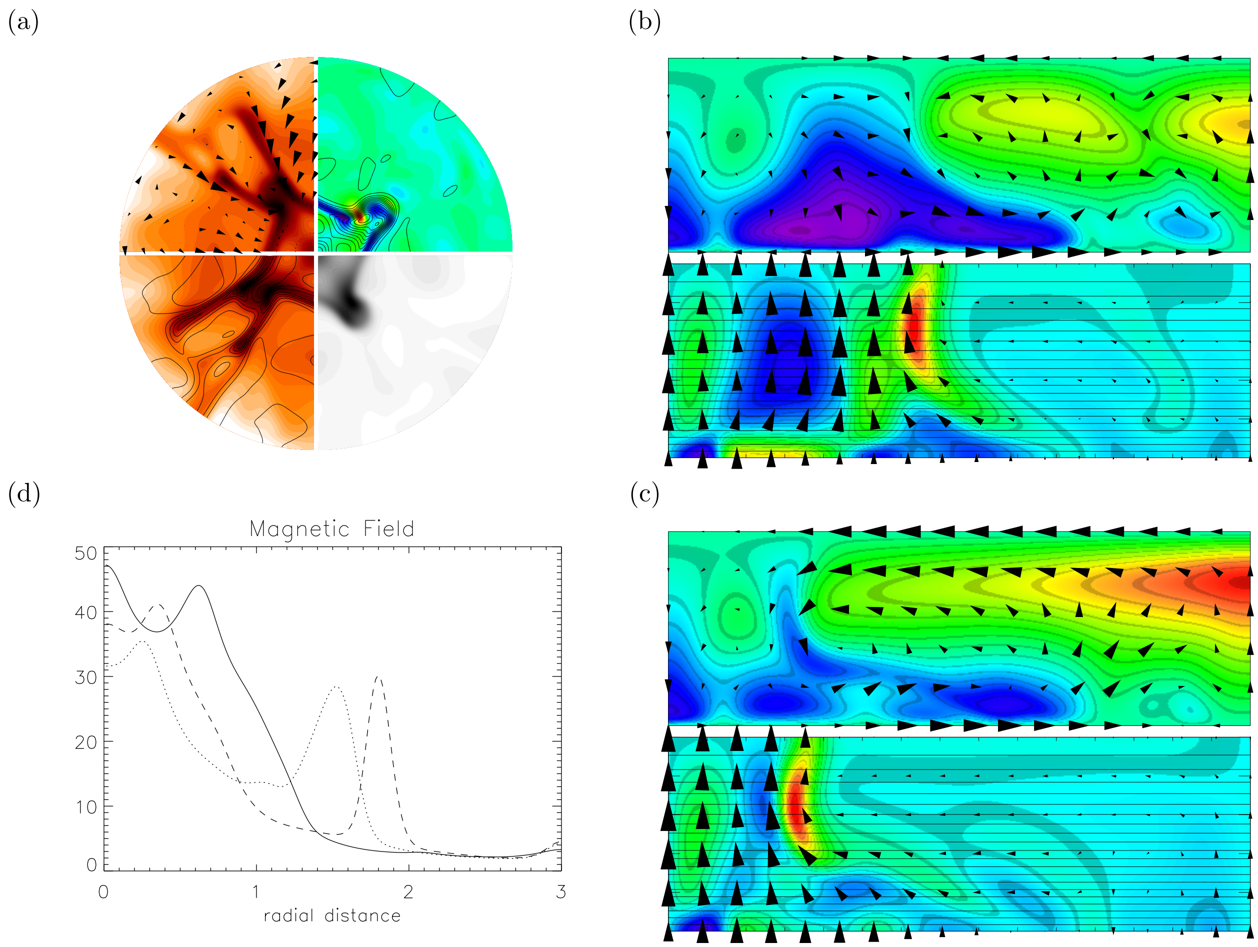


Fig. 11.— Nonlinear solution with $Q = 32$ and $M_\phi = 4$ at time 1208.72. This simulation was initialised from the numerical run depicted in Figure 4, with a data set taken at time 799.42. (a) shows the horizontal midplane with $\max|\mathbf{B}| = 12.2$, and the two vertical planes are sampled at (b) $\phi = 35°$ with $\max|\mathbf{B}| = 15.0$ and (c) $\phi = 70°$ with $\max|\mathbf{B}| = 9.9$. (d) represents the radial profile of the magnetic field strength $\sqrt{Q}|\bar{B}|$ with $|\bar{B}|$ defined in (12), at three different times. The solid line is at time 1208.72, the dotted line at time 1011.39 and the dashed line at time 800.23. The diagnostics for the horizontal plane (a) are the same as in Figure 4 and for the two vertical planes as in Figure 2(a). The physics in the two vertical planes are as follows: (b) has $\min(\beta) = 17.2$, $\max(\beta) = 7.6 \times 10^{11}$, $\min(j_\phi) = -44.4$, $\max(j_\phi) = 68.6$, $\min(\tilde{T}) = 0.98$, $\max(\tilde{T}) = 1.01$; (c) has $\min(\beta) = 27.1$, $\max(\beta) = 2.5 \times 10^{11}$, $\min(j_\phi) = -45.7$, $\max(j_\phi) = 77.7$, $\min(\tilde{T}) = 0.98$, $\max(\tilde{T}) = 1.01$. In this solution max(Mach number) $= 0.03$ in (b) and 0.09 in (c).

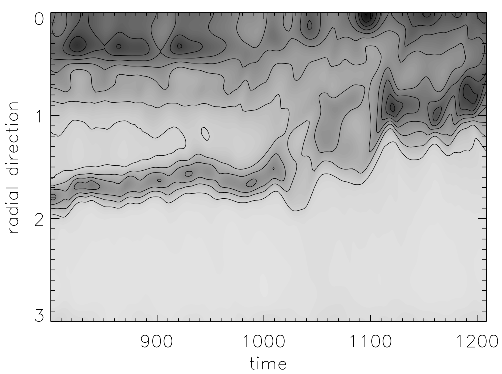


Fig. 12.— The time evolution of the effective magnetic field $\sqrt{Q}|\bar{B}|$, with $|\bar{B}|$ defined in (12), for $Q = 32$ and $M_\phi = 4$. Black represents $\max(\sqrt{Q}|\bar{B}|) = 65.5$ and white zero. The central axis is along the top edge of the panel. Three instances from this data set are plotted in Figure 11(d). The coalescing of the magnetic flux at the central axis is clearly visible.

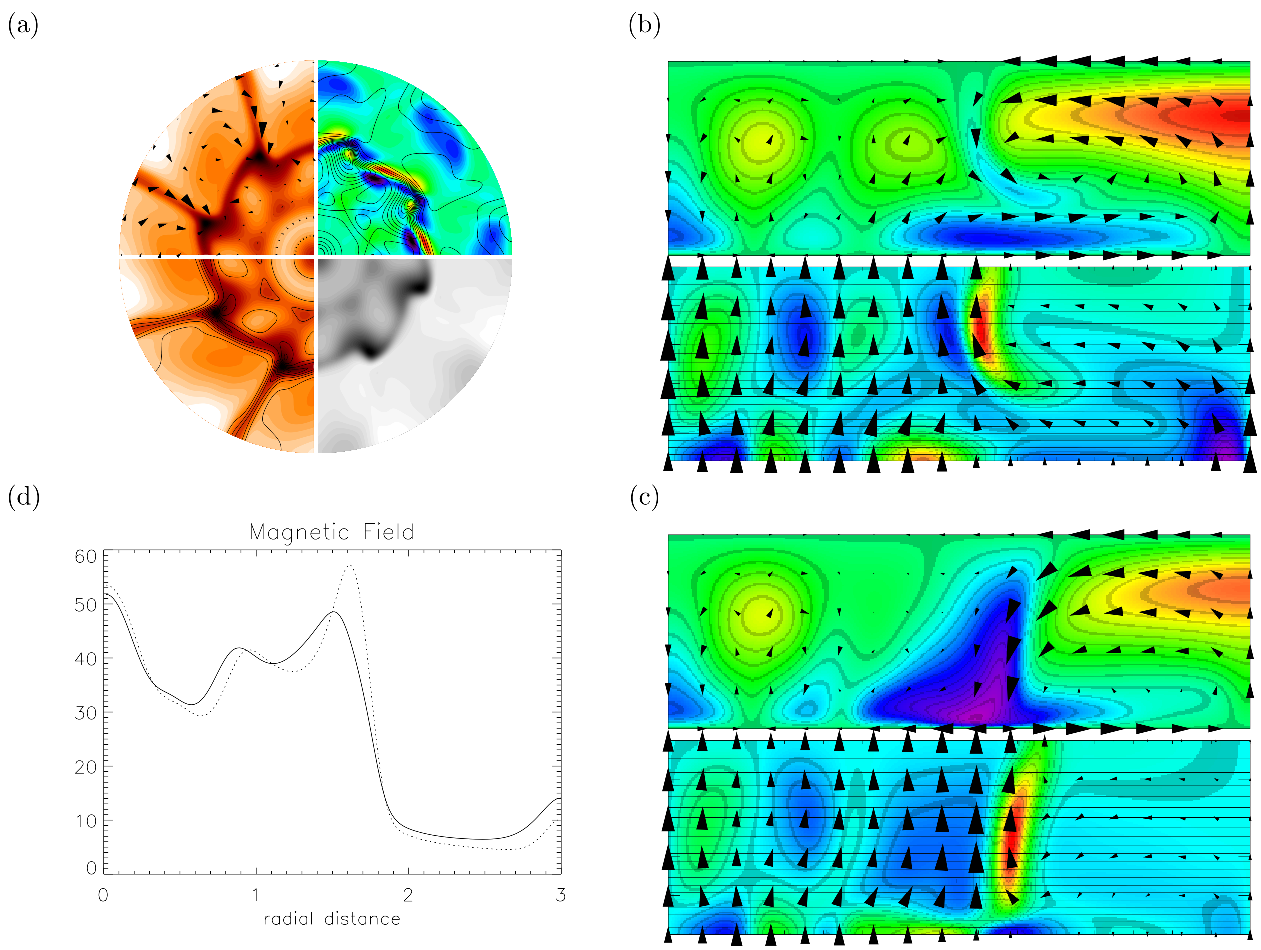


Fig. 13.— Nonlinear solution with $Q = 250$ and $M_\phi = 4$ at time 901.03. This simulation was initialised from the numerical run depicted in Figure 7, with a data set taken at time 800.83. The solution retains its $M_\phi = 8$ structure. (a) shows the horizontal midplane with $\max|\mathbf{B}| = 4.6$, and the two vertical planes are sampled at (b) $\phi = 5^\circ$ with $\max|\mathbf{B}| = 4.2$ and (c) $\phi = 30^\circ$ with $\max|\mathbf{B}| = 6.4$. (d) represents the radial profile of the magnetic field strength $\sqrt{Q}|\bar{B}|$ with $|\bar{B}|$ defined in (12), at two different times. The solid line is at time 901.03, the dotted line at time 801.64. The diagnostics for the horizontal plane (a) are the same as in Figure 4 and for the two vertical planes as in Figure 2(a). The physics in the two vertical planes are as follows: (b) has $\min(\beta) = 27.2$, $\max(\beta) = 1.2 \times 10^6$, $\min(j_\phi) = -25.5$, $\max(j_\phi) = 28.6$, $\min(\tilde{T}) = 0.98$, $\max(\tilde{T}) = 1.01$; (c) has $\min(\beta) = 13.5$, $\max(\beta) = 2.8 \times 10^9$, $\min(j_\phi) = -22.4$, $\max(j_\phi) = 48.5$, $\min(\tilde{T}) = 0.98$, $\max(\tilde{T}) = 1.01$. In this solution max(Mach number) $= 0.07$ in (b) and 0.06 in (c).